**Problem 1**

(a)



Figure1: Plot the density function f(x)

(b)



Figure 2: Histogram of the values attain by Markov Chain

Comment:

From Figure 1&2, we can see the histogram of the values attained by Markov chain is of the same shape (two peaks) as the density function f(x), only different for the scale. This Markov chain is a good way to simulate f(x).

(c)

The estimate for the value of E(x) and Var(x) using the Markov chain:

exp\_x =

0.8410

var\_x =

0.1365

Code:

clear all;

f0 = inline('(x.^2).\*abs(sin(pi.\*x.^1)).\*exp(-x.^3)','x')

c=quad(f0,0,5);

f=inline('(x.^2).\*abs(sin(pi.\*x.^1)).\*exp(-x.^3)/d','x','d');

figure(1); % plot the density function f(x)

t=0:0.01:5;

N=length(t);

plot(t,f(t,c));

title('function of f(x)');

K=5000;

x=zeros(1,K);

x(1)=0.5; %initial

for k = 2:K

y = exprnd(1);

rho = min((f(y,c)\*exp(-x(k-1)))/(f(x(k-1),c)\*exp(-y)),1);

U = rand;

x(k) = y\*(U < rho)+ x(k-1)\*(U > rho);

exp\_x(k) = mean(x);

var\_x(k) = var(x);

end

exp\_x = mean(x);

var\_x = var(x);

h=histc(x,t)/K;

figure(2);

bar(t,h);

hold on;

plot(t,f(t,c)/sum(f(t,c)),'r','linewidth',3);

**Problem 2**

When y=6, the density function become:

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I use the proposal density N(5,4), which yield very good result, but I tried another proposal density Exp(1), this also can give you similar result after more simulation steps(see the commented code).

I choose 4 different starting point: -1, 1, 5,9

**Starting point X(1)=-1:**



Figure 3: Histogram of the values attain by Markov Chain with staring point -1



Figure 4: Evolution of the chain with staring point -1(first graph is the whole 5000 points, second graph is only for 200 points)

Using the Markov Chain to estimate the posterior mean and variance with staring point x(1)=-1:

post\_mean =

5.5605

post\_var =

1.8883

**Starting point X(1)= 1:**



Figure 3: Histogram of the values attain by Markov Chain with staring point 1



Figure 4: Evolution of the chain with staring point 1

Using the Markov Chain to estimate the posterior mean and variance with staring point x(1)=1:

post\_mean =

5.4973

post\_var =

1.9664

**Starting point X(1)= 5:**



Figure 3: Histogram of the values attain by Markov Chain with staring point 5



Figure 4: Evolution of the chain with staring point 5

Using the Markov Chain to estimate the posterior mean and variance with staring point x(1)=5:

post\_mean =

5.5166

post\_var =

1.9972

**Starting point X(1)=9:**



Figure 3: Histogram of the values attain by Markov Chain with staring point 9



Figure 4: Evolution of the chain with staring point 9

Using the Markov Chain to estimate the posterior mean and variance with staring point x(1)=9:

post\_mean =

5.5523

post\_var =

1.9871

Comment:

As you can see, from different starting points, we get the similar evolution chain after few steps.

The posterior means and variances are very close to each other although the different starting points.

Code:

clear all; close all;

f0=inline('exp(-((x.^1.-5.).^2)/8).\*(exp(-abs(x.^1.-6.)).^(.5))');

c=quad(f0,-5,10);

f=inline('exp(-((x.^1.-5.).^2)/8).\*(exp(-abs(x.^1.-6.)).^(.5))/d','x','d');

t=-1:.01:11;

N=length(t);

K=5000;

x=zeros(1,K);

% try different starting points

%x(1)=-1;

%x(1)=1;

% x(1)=5;

x(1)=9;

for k=2:K

y = normrnd(5,2);

rho=min(f(y,c)\*normpdf(x(k-1),5,2)/(f(x(k-1),c)\*normpdf(y,5,2)),1);

% y=exprnd(1);

% rho = min((f(y,c)\*exp(-x(k-1)))/(f(x(k-1),c)\*exp(-y)),1);

U=rand;

x(k) = y\*(U < rho)+ x(k-1)\*(U > rho);

end

h=histc(x,t)/K;

figure(3);

bar(t,h);

hold on;

plot(t, f(t,c)/sum(f(t,c)),'r','linewidth',3);

% Show the evolution

figure(4);

subplot(2,1,1);

plot(x);

subplot(2,1,2);

% Only show the evolution of first 200 points

plot(x(1:200));

% estimate the posterior mean and variance

post\_mean=sum(x)/K;

post\_var=sum((x-post\_mean).^2)/K;